Robust Fuzzy Digital PID Control for Uncertain Dynamic Systems with Time Delay

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Abstract—In this paper, a robust fuzzy digital PID control strategy, via multiobjective genetic algorithm, based on the gain and phase margins specifications, with applications to uncertain dynamic systems with time delay, is proposed. A mathematical formulation based on the gain and phase margins, the fuzzy model and PID digital controller structures, and the robust stability of the uncertain dynamic system with time delay, is deduced. A multiobjective genetic strategy is defined to tune the fuzzy controller parameters, so the gain and phase margins specified to the fuzzy control system are get. It is proposed of deduced. A multiobjective genetic strategy is defined to tune model and PID digital controller structures, and the robust formulation based on the gain and phase margins, the fuzzy dynamic systems with time delay, is proposed. A mathematical and phase margins specifications, with applications to uncertain strategy, via multiobjective genetic algorithm, based on the gain theorem enunciates that the robust PID controller in the i-th uncertainties and dynamics of the plant to be controlled. The reference and the gain and phase margins keeping closed of the efficiency of the proposed methodology through tracking of the stability for all linear models. Experimental results shown the rule of the fuzzy robust digital PID controller guarantees the controller takes account of certain classes of parametric such as nonlinearities, uncertainties, parametric variations, complex, taking into account structural and dynamic features engineers need to deal with industrial plants increasingly complex, taking into account structural and dynamic features such as nonlinearities, uncertainties, parametric variations, time delay, among others, several methods of robust control has been proposed, allowing in their formulation the use of constraints and performance requirements [1][2][3][4][5][6]. In [7], the stabilization of a discrete time robust control system based on reference model, is achieved. In [8], the design of robust control for perturbed systems, is presented. In [9], a fuzzy PID control to improve the stability of hydraulic position servo system, is proposed. Furthermore, fuzzy controllers have been a good alternative for control of complex dynamic systems, once that the fuzzy structure based on rules are able to treat uncertainties, nonlinearities and time delay problems [10][11][12][13]. In this paper, a model based robust fuzzy digital PID control strategy, via multiobjective genetic algorithm, based on the gain and phase margins specifications with applications to uncertain dynamic systems with time delay, is proposed.

II. STRATEGY FOR ROBUST FUZZY DIGITAL PID CONTROLLER DESIGN

A. Tuning formulas for model based control via gain and phase margins specifications

The Takagi-Sugeno (TS) fuzzy inference system to be used as model of the uncertain dynamic system, presents the i-th rule given by:

\[ R^{(i)}: \text{IF } \tilde{y}(k-1) \text{ IS } F_{k|y(k-1)}^{i} \text{ THEN} \]

\[ G^p_p(z) = \frac{K^i_1 z + K^i_2}{a^i z^2 + b^i z + c^i} \]

where \(a^i, b^i, c^i, K^i_1,\) and \(K^i_2\) are the parameters to be estimated by least square algorithm. The variable \(\tilde{y}(k-1)\) belongs to fuzzy set \(F_{k|y(k-1)}^{i}\) with a value \(\mu_{F}^{i}_{k|y(k-1)}\) defined by a membership function \(\mu_{F}^{i}_{y(k-1)} : R \rightarrow [0,1]\), with \(\mu_{F}^{i}_{k|y(k-1)} \in \mu_{F}^{1}_{k|y(k-1)} , \mu_{F}^{2}_{k|y(k-1)} , \mu_{F}^{3}_{k|y(k-1)} , \ldots \)

\[ \mu_{F}^{j}_{p_{y(k-1)}}(\tilde{y}(k-1)) \]

where \(p_{y(k-1)}\) is the partitions number of the universe of discourse related to linguistic variable \(\tilde{y}(k-1)\) [14] [15][16]. The TS fuzzy digital PID controller, according to Parallel Distributed Compensation (PDC), presents the j-th rule given by:

\[ R^{(j)}: \text{IF } \tilde{y}(k-1) \text{ IS } F_{k|y(k-1)}^{j} \text{ THEN} \]

\[ G^j_p(z) = \frac{\alpha^j z^2 + \beta^j z + \gamma^j}{z^2 + z} \]

where:

\[ \alpha^j = K^j_p + \frac{K^j_T}{2} + \frac{K^j_D}{T} \quad (1) \]

\[ \beta^j = \frac{K^j_T}{2} - K^j_p - \frac{2K^j_D}{T} \quad (2) \]

\[ \gamma^j = \frac{K^j_D}{T} \quad (3) \]
where \(K_p, K_i, \) and \(K_d\) are proportional, integral, and derivative fuzzy controller gains, and \(T\) is the sample time, respectively.

In the direct path of closed-loop control system, considering the TS fuzzy model, the time delay \(z^{-n}\), and the fuzzy digital PID controller, it has:

\[
G_c(\tilde{y}(k-1), z)G_p(\tilde{y}(k-1), z) = \sum_{i=1}^{l} \sum_{i=1}^{l} \gamma_i(\tilde{y}(k-1)) \times \gamma_j(\tilde{y}(k-1)) \times \left( \frac{\alpha_i z^2 + \beta_j z + \gamma_i}{z^2 + z} \right) \times \frac{K_1}{z} + \frac{K_2}{z^2} + b'z + c'z^{-n}
\]

The gain and phase margins of the fuzzy control system are given by:

\[
\arg[G_c(\tilde{y}(k-1), e^{j\omega_p})G_p(\tilde{y}(k-1), e^{j\omega_p})] = -\pi
\]

\[
A_m = \frac{1}{|G_c(\tilde{y}(k-1), e^{j\omega_p})G_p(\tilde{y}(k-1), e^{j\omega_p})|}
\]

\[
|G_c(\tilde{y}(k-1), e^{j\omega_i})G_p(\tilde{y}(k-1), e^{j\omega_i})| = 1
\]

\[
\phi_m = \arg[G_c(\tilde{y}(k-1), e^{j\omega_i})G_p(\tilde{y}(k-1), e^{j\omega_i})] + \pi
\]

where the gain margin is given by (5) and (6), and the phase margin is given by (7) and (8), respectively. The \(\omega_p\) is called phase crossover frequency and \(\omega_i\) is called gain crossover frequency [17][5].

B. Robust Stability Analysis

For the robust fuzzy PID digital controller design, based on the gain and phase margins specifications, the following Axiom and Theorems are proposed:

**Axiom**: The linear sub-models \(G_p(z)\) \([i=1,2,...,l]\) of the plant, are necessarily of minimum phase, i.e., all poles of the characteristic equation are placed inside of the unit circle of the \(z\)-plane.

**Theorem 1**: Each robust PID sub-controller, \(G_p(z)\) \([i=1,2,...,l]\), guarantees the gain and phase margins specifications for the linear sub-model,\(G_p(z)\) \([i=1,2,...,l]\) with \(i = j\), of the plant to be controlled.

**Proof**: The proof of this theorem is not shown since there is no space enough in this paper.

**Theorem 2**: Each robust PID sub-controller \(G_p(z)\) \([i=1,2,...,l]\) guarantees the stability for all linear sub-models \(G_p(z)\) \([i=1,2,...,l]\) of the nonlinear plant to be controlled.

**Proof**: The proof of this theorem is not shown since there is no space enough in this paper.

C. Multiobjective Genetic Strategy for Controller Tuning

The GA proposed in this paper to optimize the parameters \(K_p, K_i, \) and \(K_d\) of fuzzy digital PID controller in the \(j\)-th rule from the gain and phase margins specifications, presents the cost function given by:

\[
Cost = |A_{mcal} - A_{mesp}| + |P_{mcal} - P_{mesp}|
\]

where \(A_{mcal}\) and \(A_{mesp}\) correspond to the gain margin computed and specified, respectively; \(P_{mcal}\) and \(P_{mesp}\) correspond to the phase margin computed and specified, respectively. The crossover between two chromosomes, used by the multiobjective genetic algorithm generates two new offspring by a simple crossover operator, which performs a weighted sum between the genes of the parents in order to generate the offspring. As follow:

\[
\text{chromosome}_1 = [p_{m1}, p_{m2}, p_{m3},...,p_{mn}]
\]

\[
\text{chromosome}_2 = [p_{p1}, p_{p2}, p_{p3},...,p_{pn}]
\]

\[
d_{new1} = \beta \times p_{mn} + (1 - \beta) \times p_{pn}
\]

\[
d_{new2} = \beta \times p_{pn} + (1 - \beta) \times p_{mn}
\]

where the terms \(p_{mn}\) and \(p_{pn}\) represent the \(n\) genes mother chromosome \((\text{chromosome}_1)\) and genes father chromosome \((\text{chromosome}_2)\) respectively, \(d_{new}\) is new offspring generated from two chromosomes and \(\beta\) is a random value between 0 and 1. The mutation operator used in this paper selects randomly a gene from the chromosome of the population and change its value to any other, within the range of possible values that the gains of the fuzzy controller can take. The best chromosome of the previous generation is kept for the next generation, which is complemented by selected parents and the result of the mutation on the offspring. The stages of evaluation, classification, partner selection, crossover, mutation, and formation of the new population are repeated at each iteration of the algorithm [18][19].

III. EXPERIMENTAL RESULTS

A. Description of the Platform for Real Time Control of a Termic Process

The platform for real time control is composed of the termic process, the software LabVIEW, the CompactRIO 9073, the analog input module NI 9219, the analog output module NI 9263, the temperature sensor LM 35 and the actuator based on C1 TCA 785. The termic process consists in an AC 220Volts monophasic toaster, with functional temperature in the interval from 25°C to 265°C. The Labview is a graphical programming language in which the supervisory system will be developed for real time analysis of the closed loop control: the temperature and input voltage signals will be received from the sensor through the data acquisition system, the fuzzy robust PID controller will be implemented to process the control signal to be sent to the termic process. The platform control system of the termic process for real time is shown in the Fig. 1.
B. TS Fuzzy Modeling for Termic Process

In the identification procedure, it was used the input and output signals of the uncertain dynamic system, as shown in Fig. 2. The time delay was estimated by computing sample cross-correlation function between the input and the output signals, resulting in a time delay of 1360 samples, considering the sample time of 17ms, corresponding to 2.312 seconds [20], as shown in Fig. 3.

In order to estimate the fuzzy sets, the FCM (Fuzzy C-Means) algorithm was implemented for 2 clusters, weighting exponent \( m=1.2 \) and tolerance \( \epsilon=0.01 \). The fuzzy sets obtained are shown in Fig. 4. From the data input and output of the uncertain dynamic system, taking into account the weights of fuzzy sets, the least squares algorithm was applied for parameter estimation of the consequent sub-models, resulting in the fuzzy model structure of the uncertain dynamic system to be controlled. However, the identified TS fuzzy model was submitted to optimization procedure by genetic algorithm to improve DC gains to sub-models. The gains obtained were \( g_1 = 0.0055 \) and \( g_2 = 0.72 \), corresponding to sub-models \( G^1_p \) and \( G^2_p \), respectively. A comparative analysis for the temporal response performance of the termic process, TS fuzzy model optimized and identified is shown in Fig. 5. The identified TS fuzzy model after optimization procedure is given by (12).

\[ R^1 : IF \text{ Temperature is } F^1, \text{ THEN} \]
\[ G^1_p(z) = \frac{0.0055 + 0.0504}{z^2 - 0.5815z - 0.4177} z^{-136} \]  
\[ R^2 : IF \text{ Temperature is } F^2, \text{ THEN} \]
\[ G^2_p(z) = \frac{0.72 - 0.1344z - 0.1334}{z^2 - 0.5921z - 0.4072} z^{-136} \]  

where:
\[ F^1(a, b) |a=70; b=150.2 \]  

\[ 1. \text{ Temperature} \leq a \]
\[ 2. \frac{(\text{Temperature} - a)^2}{a - b} \leq \text{ Temperature} \leq \frac{a + b}{2} \]
\[ 3. \frac{(b - \text{Temperature})^2}{b - a} \leq \text{ Temperature} \leq b \]

and \( F^2 = 1 - F^1 \). From the multiobjective genetic strategy proposed in this paper, specifying the appropriate gain and phase margins for
the fuzzy control system, and considering the fuzzy model of
the uncertain dynamic system with time delay of $n = 136$, the
parameters of the fuzzy digital PID controller were obtained,
according to table. I. It can be seen the efficiency of the
proposed methodology in the model based PID controller
design, since the gain and phase margins obtained of the fuzzy
control system keeping close of the specified ones. The Bode
Diagram of $G_1^1$ and $G_1^2$ are shown in Fig. 6 and Fig. 7,
respectively.

**TABLE I**

| Sub-model | $|\Delta_{inc},\Delta_{inc}|$ | PID Gains ($K_P, K_I, K_D$) |
|------------|-----------------------------|-----------------------------|
| $G_1^1$    | (2.5, 60°)                  | (2.9643, 63.8115°)           |
| $G_1^2$    | (2.5, 60°)                  | (2.5936, 62.6088°)           |

The multiobjective genetic algorithm used the following
parameters: 300 generations, random initial population of 100
chromosomes, each chromosome comprising three genes, the
selection rate of 50% and the mutation rate of 12%. The
performance of the multiobjective genetic strategy, to minimize
the multiobjective cost function according to the number of
generations, is shown in Fig. 8. The parameters of fuzzy digital
PID controller obtained by the multiobjective genetic strategy
proposed was compared to Ziegler-Nichols Method [21]. The
temporal response performance of the proposed methodology
is shown in Fig. 9. The initial condition for temperature set
point was of 100°C. The changing to 150°C in the temperature
set point was applied at 425 seconds. A gain variation was
applied to the termic process at 212 seconds.

**IV. CONCLUSION**

In this paper, a methodology for robust fuzzy digital PID
control design of the uncertain dynamic systems with time
delay, was proposed. The control scheme was efficient to
ensure, through the multiobjective genetic approach, since gain
and phase margins specified were get and keeping closed of the specified ones - despite the genetic algorithm application requires time and computational effort - as well as the tracking of the reference trajectory, even if with disturbance. The stability and high performance in the time has been guaranteed by the proposed TS fuzzy robust digital PID controller as compared with other controller from literature.

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